Neutron Density Distributions of Nickel Isotopes Analyzed in Terms of Relativistic Impulse Approximation

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Abstract:
Recent relativistic mean-field calculations have provided nuclear distributions many isotopes whose neutron numbers are much larger than their atomic numbers. The author calculates observables of proton elastic scattering from some of those unstable isotopes and discusses relations between observables and nuclear distributions of such unstable nuclei, especially nickel isotopes ^{58,60,62,64}Ni. The calculations are based on relativistic impulse approximation (RIA) at incident proton energies from 200 through 500 MeV where predictions of RIA have been shown to provide good agreement with experimental data.

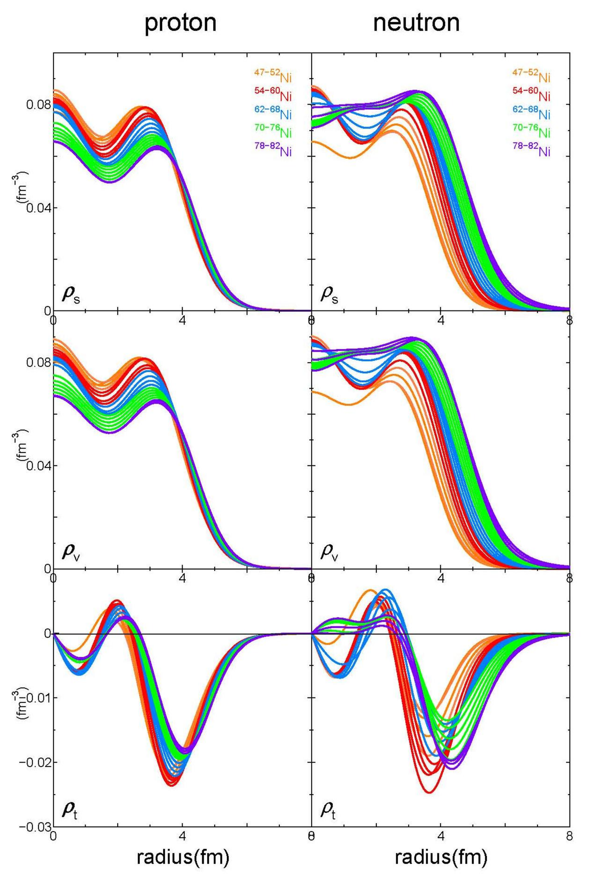
Relativistic Mean Filed Density of Nickel Isotopes

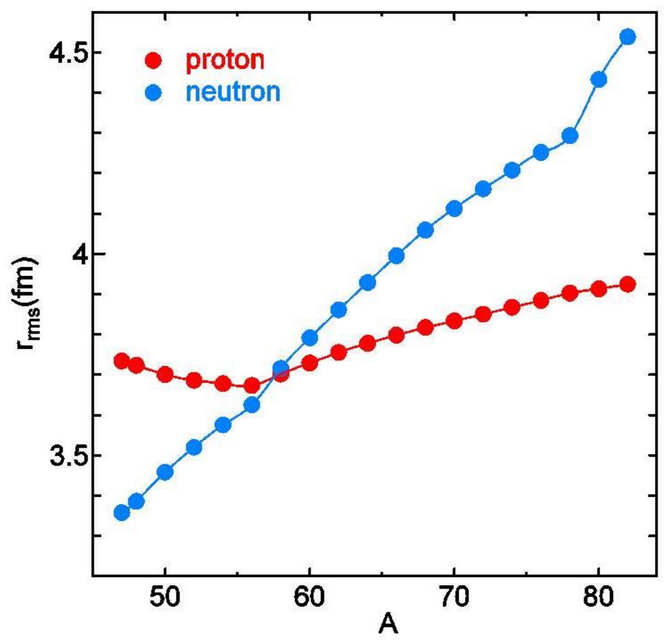
• density matrices:

$$\hat{\rho}(\mathbf{q}) = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\, \hat{\rho}(\mathbf{r})$$

$$\hat{\rho}(\mathbf{k},\mathbf{q}) = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\, \hat{\rho}(\mathbf{r}(k))$$

$$\hat{\rho}(\mathbf{r}) = \rho_S(\mathbf{r}) \quad \text{← scalar}$$
$$+\gamma^0\rho_V(\mathbf{r}) \quad \text{← vector}$$
$$-\frac{i\boldsymbol{\alpha}\cdot\hat{\mathbf{r}}}{2}\rho_T(\mathbf{r}) \quad \text{← tensor}$$





root-mean square radius (fm)

Isotope	n(RMF)	p(RMF)	p(G)	p(FB)
^{58}Ni	3.7160	3.7020	3.6763	3.6732
^{60}Ni	3.7916	3.7523	3.7016	3.6856
^{62}Ni	3.8613	3.7555	3.7394	3.7063
^{64}Ni	3.9289	3.7782	3.7589	3.7308

charge radius

RIA Calculations: Proton Elastic Scattering from ^{58}Ni

Relativistic Impulse Approximation (RIA)

• optical potentials

Dirac equation for a projectile proton scattering from a target nucleus in the momentum space

$$\left(\gamma^0 E - \boldsymbol{\gamma}\cdot\mathbf{p}' - m\right)\psi(\mathbf{p}') - \frac{1}{(2\pi)^3}\int d^3p\, \hat{U}(\mathbf{p}',\mathbf{p})\,\psi(\mathbf{p}) = 0$$

relativistic analog of non-relativistic multiple scattering theory, optical potential in coordinate space:

$$\hat{U}(\mathbf{r}) = <\Phi|\sum_i t_i|\Phi> + \sum_{i\neq j}\sum_j <\Phi|t_i \bar{G} t_j|\Phi> - \frac{A-1}{A}<\Phi|\sum_i t_i|\Phi>\bar{G}<\Phi|\sum_j t_j|\Phi>$$

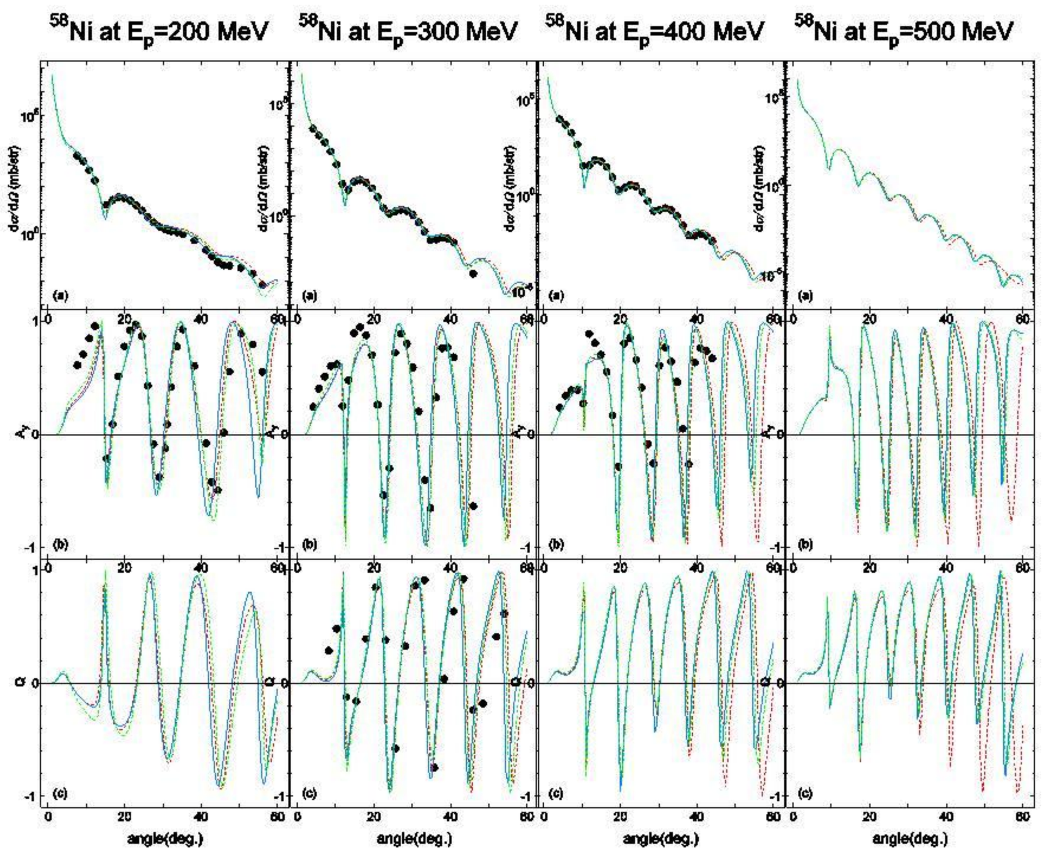
1st order term · 2nd order term

the generalized RIA optical potential in the momentum space for the 1st order term

$$\hat{U}(\mathbf{p}',\mathbf{p}) = -\frac{1}{4}\mathrm{Tr}\left\{\int\frac{d^3k}{(2\pi)^3}\hat{M}_{pp}(\mathbf{p},\mathbf{k}-\frac{\mathbf{q}}{2}\to\mathbf{p}',\mathbf{k}+\frac{\mathbf{q}}{2})\,\hat{\rho}_p(\mathbf{k},\mathbf{q})\right\} - \frac{1}{4}\mathrm{Tr}\left\{\int\frac{d^3k}{(2\pi)^3}\hat{M}_{pn}(\mathbf{p},\mathbf{k}-\frac{\mathbf{q}}{2}\to\mathbf{p}',\mathbf{k}+\frac{\mathbf{q}}{2})\,\hat{\rho}_n(\mathbf{k},\mathbf{q})\right\}$$

optimal factorization : $(\mathbf{k}=0)$ simple tp-form

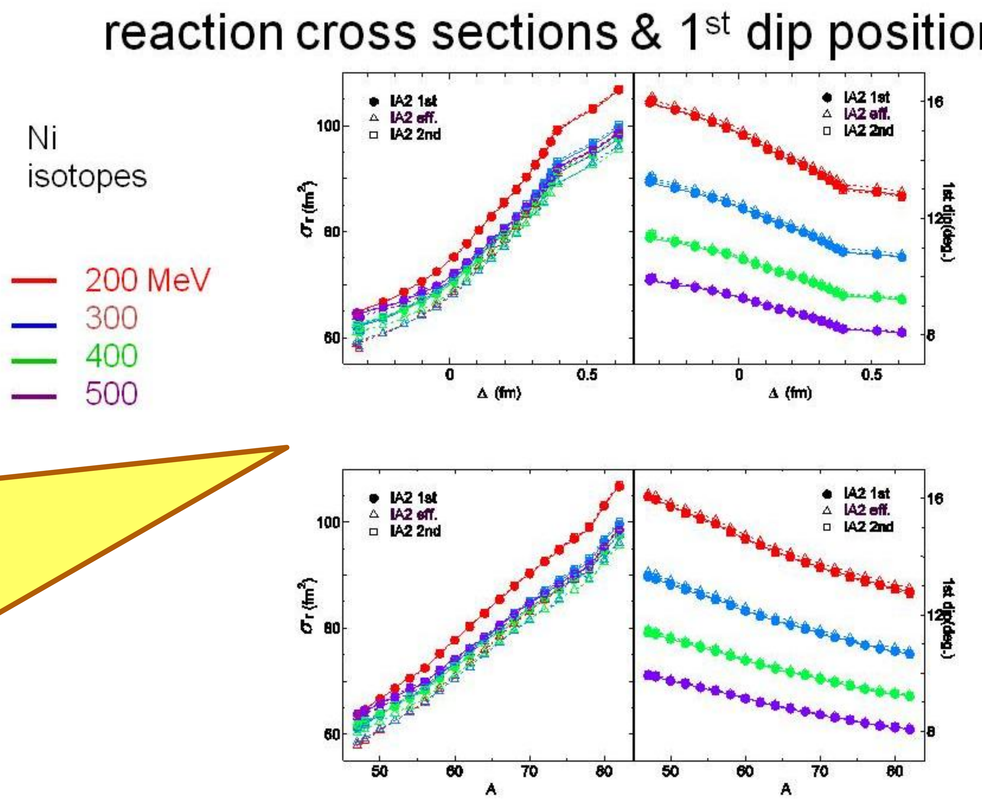
$$\hat{U}(\mathbf{p}',\mathbf{p}) = -\frac{1}{4}\mathrm{Tr}\left\{\hat{M}_{pp}(\mathbf{p},-\frac{\mathbf{q}}{2}\to\mathbf{p}',\frac{\mathbf{q}}{2})\,\hat{\rho}_p(\mathbf{q})\right\} - \frac{1}{4}\mathrm{Tr}\left\{\hat{M}_{pn}(\mathbf{p},-\frac{\mathbf{q}}{2}\to\mathbf{p}',\frac{\mathbf{q}}{2})\,\hat{\rho}_n(\mathbf{q})\right\}$$



two restricted observables
• reaction cross section
• 1st dip position of the differential cross section

contributions of medium and multiple scattering effects are not significant in both reaction **cross section** and the first dip position of differential cross section in energies larger than 300 MeV.

reaction cross sections & 1st dip position



A Model Analysis

assumed neutron distribution

normalized by $A-Z=4\pi\int\rho(r)r^2dr$

$$\rho(r) = \frac{\rho_0}{1+\exp\{(r-r_0)/a\}}$$

radial parameter · diffuseness parameter

Fourier transformation

$$\rho(q) = 4\pi\int_0^\infty j_0(qr)\frac{\rho_0}{1+e^{(r-r_0)/a}}r^2\,dr \approx \frac{4\pi\rho_0}{q^3}\frac{\pi q a}{\sinh(\pi q a)} \times \left\{\begin{array}{l}\pi q a\cdot\coth(\pi q a)\sin(q r_0)\\ -q r_0\cdot\cos(q r_0)\end{array}\right\}$$

two observables are related to the different kind of information on the density distribution.

1. differential cross section

the 1st Born approx. t-ρ form

$$\frac{d\sigma}{d\Omega} = |f_B(\theta)|^2 = \left(\frac{\mu}{2\pi\hbar}\right)^2|t(q)|^2\rho^2(q)$$

1st dip position ⟺ $\rho(q)=0$

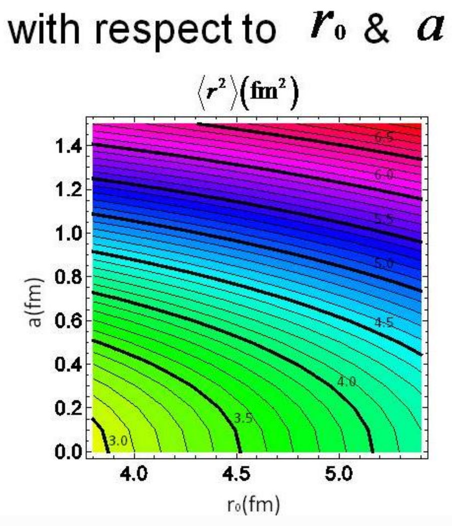
2. mean-square radius ∝ reaction cross section

$$\langle r^2\rangle = 4\pi\int r^2\rho(r)\,r^2dr\Big/4\pi\int\rho(r)\,r^2dr = \frac{1}{5}\left[7(\pi a)^2+3r_0^2\right]$$

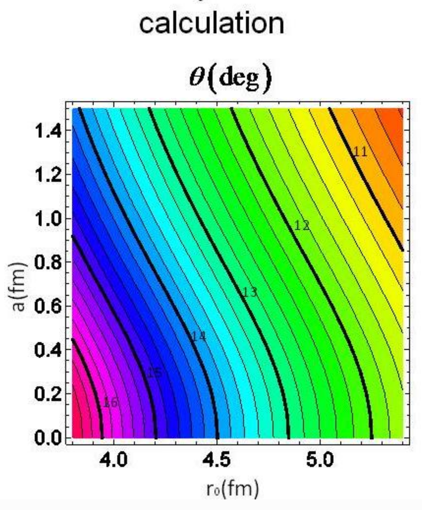
analytic function of the parameters

two parameters are determined by the two observables.

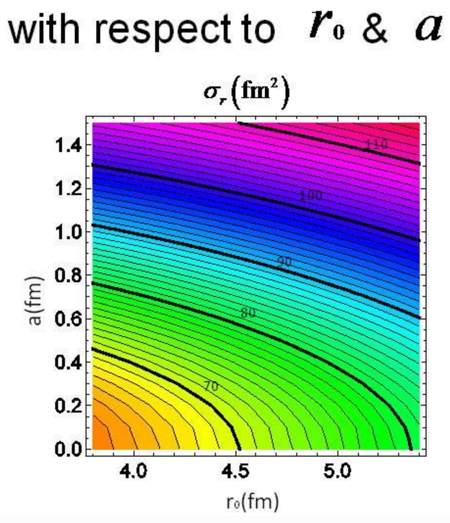
contour map of msr & dip with respect to r_0 & a



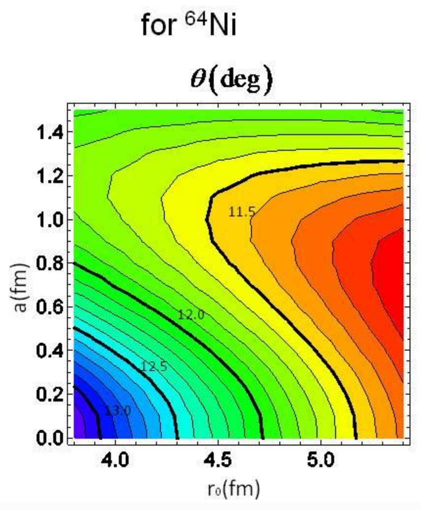
analytic calculation



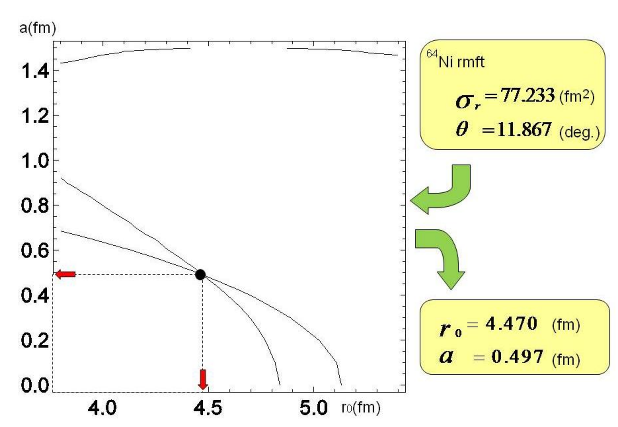
contour map of rcs & dip with respect to r_0 & a



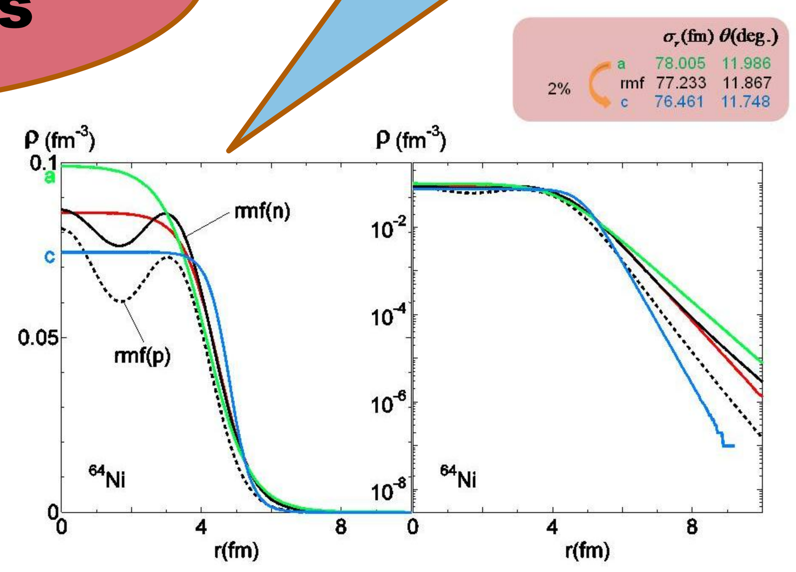
observables for ^{64}Ni



How to determine the parameters

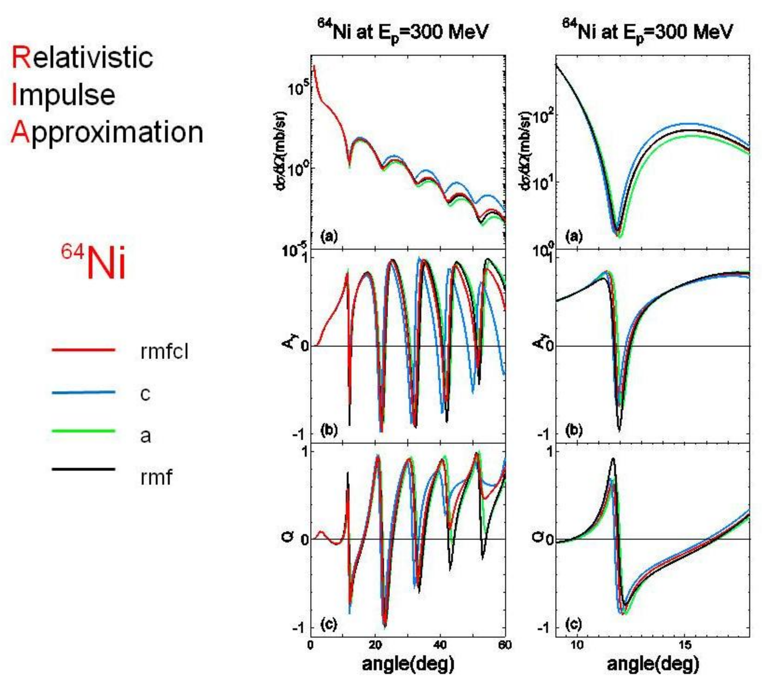


obtained density distributions



Relativistic Impulse Approximation

^{64}Ni



	rmsr (fm)	Δ(fm)
rmfcal	3.9244	-0.0045
a	3.9490	0.0201
c	3.8962	-0.0327
RMFT	3.9289	0

references
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2. K.Kaki Int.J.Mod.Phys. E13 (2004) 787-799
3. K.Kaki, Phys.Rev. C79 (2009) 064609

Summary:
Observables of proton elastic scattering from nickel isotopes are calculated with RIA and RMFT. The results for stable ^{58}Ni target at 300-500 MeV show good agreement with experimental data. As for unstable nickel isotopes, a model analysis in which the Woods-Saxon distribution is assumed for neutron density provides a prescription for determination of two parameters of assumed distribution in terms of two restricted observables: reaction cross section and the 1st dip position of differential cross section. Calculations for 64Ni target at 300 MeV demonstrates that the parameters determined with those observables given by RMFT density reproduce very well the profile of RMFT distribution, especially in the surface region.